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# MODIFIED GOEL - OKUMOTO MODEL FOR ESTIMATING SOFTWARE RELIABILITY

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**Abstract**: Software Reliability Growth Model (SRGM) is one of the fundamental techniques used to assess reliability of software quantitatively. Software reliability growth model is required to have a good performance in terms of goodness of fit, predictability, and so forth. A number of models have been proposed during the past decades for reliability analysis of a software system. But many of them are insufficient to correctly analyze the exact reliability of the software. Moreover the calculation of the time period up to which the testing has to be done before the release of software for public use is not possible to predict through these models. Therefore in this chapter we have presented a Modified Goel-Okumoto Model through which a threshold value is calculated which will provide the software development team with the minimum time period up to which the testing has to be done and after that the software can be released for public use. But the applicability of SRGM highly depends upon the correct estimation of parameters. Estimation of model parameters is usually different according to different estimation methods. In this paper we have also will summarized about the parameters and the significance of these parameters on the predictability of reliability by the Software Reliability Growth Models (SRGMs).

**Keywords: -** Goel-Okumoto Model, Software Reliability Growth Models, Roundness factor.

**Introduction:** Any software reliability growth model (SRGM) can be considered to be a mathematical expression which fits the experimental data. It may be obtained by simply observing the overall trend of reliability growth. But some of the models can also be obtained analytically by making some assumptions about the software testing and debugging. Some of the assumptions are to keep the analysis tractable; other can be more fundamental in nature and can constitute modeling of the testing and debugging process itself. Goel discussed the applicability and limitations of software reliability growth models during the software development life cycle in [2].The parameters of the model also have an important role in the interpretation of the testing process and the reliability of the software. A clear understanding of the underlying parameters gives us a valuable insight into the process [4].

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1. If we know how a parameter arises, it can be estimated even before testing begins. Such a priori values when are estimated using past experience, it can be used to do the preliminary planning and resource allocation before testing begins[5].
2. The use of SRGMs suggests that in the beginning of testing, the initial test data yields unstable parameter values. In such cases, values can be estimated using static information, which can serve as a check. These parameter values can also be used to stabilize the projections adding to the information obtained by the dynamic defect detection data.
3. Many a times iterative techniques can be used to estimate the parameter values. The values obtained depend on the initial estimates that are required for numerical computation. Use of a priori values as the initial estimate would initiate the search and lead closer to the values sought.
4. Parameters that have an interpretation symbolize the testing and debugging process quantitatively. Their values can give us an understanding of the process. They may help at reveal about how the inherent defect density can be reduced or how testing can be made more efficient.

This chapter examines the parameters of the exponential models. A new interpretation of the parameters of the exponential model is presented.

**Implication of Modified Goel-Okumoto Model**

All software reliability growth models are approximations of real processes, thus none can be considered perfect, but Goel-Okumoto model (Exponential) is found to be applicable for a variety of software. Therefore this model has been chosen for further study.

The Goel-Okumoto Model predicts the predicted error with the help of roundness factor and the time of error reporting. But it is insufficient to further analyze the reliability of the software. Therefore, modification in the model has been done which includes a new variable, the threshold value (α). The threshold value (α) gives the minimum time after which the software is considered to be reliable or the maximum time up to which the software is highly vulnerable and is not ready for release for public use. The **modified Goel-Okumoto** Model is presented as follows:

$\mu(t) = E_E\,(1 - e^{-bt})$ (Original Goel-Okumoto Model)

$\mu'(t) = E^E(1-e^{-b'(t-\alpha)})$ where $b'>0$

(Modified Goel-Okumoto Model) (1)

$\mu'(t)$ = Mean value function or the predicted number of bugs at time t

$E_E$ = Expected total number of defects in the code

**b' = Mean value (Roundness factor)**

t = Calendar time/ execution time/ number of test runs

**α = Threshold value**

Further the reliability of the software can be estimated by the Reliability Factor (RF). Further in this study reliability factor at two instances is done for comparison. First at the threshold time and at final testing time. It is found out that the reliability at the two instances is not varying to a larger extent. Which indicates that the after the threshold time the software is approaching reliability and is ready for public dispatch.

Parameter estimation is a discipline that provides methods for efficient use of data for aiding in mathematical modeling of phenomenon and estimations of constants appearing in these models [1]. Most of the parameter estimation can be related to four optimization problems [6]:

- Criterion: the choice of the best function to optimize (maximize or minimize)
- Estimation: the choice of the best function to optimize the chosen function
- Design: Optimal implementation of the chosen method to obtain the best parameters estimates.
- Modeling: The determination of the mathematical model which best describes the system from which data are measured, including the model of the error processes.

Further, the reliability of the software can also be estimated by the alternate quantities of the failure data. Typically, software reliability predictions take into account factors such as the size and complexity of a program. These predictions can be used to analyze the reliability of the software.

**Significance of the Parameters in Modified Goel-Okumoto Model**

Here we derived a method to evolve the parameters (EE, b' and α) of this model.

**Total Expected Errors ($E_E$)**

It has been observed that for a specific development environment and for the same software development team and for the same development/testing period, the error density encountered is same [4]. $E_E$ is the number of errors which is expected to be in the software when the testing period starts. As the testing continues errors are detected and debugged. In this model it is considered that as errors are debugged new errors are not introduced. $E_E$ is computed on various factors which can affect the software development i.e the quantity which has to be assumed is influenced by several independent factor. Some of the factors which are discussed below are:

- Phase factors
- Programming Team Factor
- Maturity factor
- Structure Factor
- Requirements volatility factor

**Mean value (Roundness factor) (b`):** Roundness is a measure of the compactness of a shape. A circle is the most compact shape, so the more compact a shape is, the more closely it resembles a circle. Roundness is a ratio and therefore a dimensionless number. The roundness factor may be calculated for any two dimensional shape as long its area and perimeter is known.

The roundness factor (b`) in SRGM depict the rate at which the error is decreasing in the software while testing or the roundness factor (b`) represent the failure rate of the software. The **modified Goel - Okumoto** model predicts the number of errors which the software might encounter. As the testing proceeds the errors goes on decreasing (it is assumed that the errors are rectified as they are encountered).

For Exponential Model (increasing form) of equation 1 it posses the following features:

- It is asymptotic in nature i.e y = a to right
- The curve passes through the coordinates (0,0)

- a is the upper limit
- The value will Increase, but bounded above by y=a

Exponential decay models of this form can model sales or learning curves where there is an upper limit. This is done by subtracting the exponential expression from one and multiplying by the upper limit. Exponential decay models of this form will increase very rapidly at first, and then level off to become asymptotic to the upper limit.

**Threshold Value (α):** The threshold value represents that time period which is considered to be the maximum time after which the software is considered to be reaching its maximum reliability. Up to the threshold time the maximum errors would have been found out and debugged (It is assumed that as no new errors are introduced while the errors are debugged). After the threshold time the software can be considered to be the main version.

**Computing the values of b and α for Modified Goel-Okumoto Model:** Computing the value of b and α, from equation 1

$\mu(t) = E_E (1-e^{-b`(t-\alpha)})$, where b>0 therefore,

$\frac{\mu(t)}{E_E} = (1-e^{-b`(t-\alpha)})$

$e^{-b`(t-\alpha)} = 1- \frac{\mu(t)}{E_E}$

Taking natural log (ln) on both sides, we have

$\ln(e^{-b`(t-\alpha)}) = \ln(1- \frac{\mu(t)}{E_E})$

$-b`(t-\alpha) = \ln(1 - \frac{\mu(t)}{E_E})$ because $\ln(e^x) = x$

$u= b`\alpha - b`t$ (4) $(u=-\ln(1-\frac{\mu(t)}{E_E}))$

$\sum u = n\alpha\, b` - b`\sum t$ (5)

$\sum tu = \alpha b` \sum t - b`\sum t^2$ (6)

Solving both equations (5) and (6)

$b` = \frac{\sum u \sum t - n\sum tu}{n \sum t^2 - (\sum t)^2}$ (7)

From equation 4

$\alpha = (u + b\,t)/ b`$ (8)

**Reliability Factor**

Reliability Factor (RF) = $1-(E_R / E_E)$ [3] (9)

$E_E$ = Total Expected Errors

$E_E$ is the total number of errors which is expected to be in the software when the testing period starts. $E_E$ is estimated depending upon various factors and has been discussed above.

$E_R = E_E$ – Actual Error detected and rectified = Residual Errors

It represents the errors which are considered to be in the software even after testing is completed with reference to the Total Expected Error.

Since $E_E$ is the maximum error which is expected in the software prior to testing. And since the residual error ($E_R$) is the difference between the Total Expected Error ($E_E$) and Actual Errors detected it can be utmost be equal to $E_E$ or less than $E_E$. On the basis of this idea the ratio, $E_R / E_E$ will be less and $1-(E_R / E_E)$ will be less than 1. This ratio gives the idea of the validity of the model and therefore $1-(E_R / E_E)$ can be considered as Reliability Factor (RF) on the basis of modified Goel-Okumoto Model.

Thus Reliability Factor (RF) is the measure for software reliability [3]. Its value will vary between 0 and 1. If RF=1, then the software under consideration is perfect, however, if RF=0, then the software is highly vulnerable. When RF approaches close to 1 then the software can be considered as reliable.

**Case Study**

**Debain Operating System:** To further analyze the behavior of b and α data set has been taken from the bug reports by Debain [8] users, which are hoisted at the website of Debain Operating System. Debain is a Computer operating system composed of software packages released as free and open source software primarily under the GNU General Public License along with other free software license [9]. The data set consists of the error reported by different users working on multicore architecture for 109 months [Appendix A].

$b` = \frac{\sum u \sum t - n\sum tu}{n \sum t^2 - (\sum t)^2}$ **(from 7)**

n = 109

$\sum tu = -3724.84$ (From Table 3)

$\sum u = -51.0252$ (From Table 3)

$\sum t = 5995$ (From Table 3)

$\sum t2 = 437635$

$(\sum t)2 = 35940025$

$b` = \frac{-51.0252 * 5995 - (109 * (-3724.84))}{109 * 437635 - (5995)^2}$

$b` = \frac{711903.634}{11762190}$

**b` = 0.060524**

$\alpha = \frac{-0.92773 + 0.060524 * 109}{0.060524} = \frac{5.669386}{0.060524}$

**α = 93.671**

**α = 94 months**

The predicted errors at 94 months are 151.472 = 151 (From Table 3)

And the actual errors found out and rectified is 171

Total Expected Error ($E_E$) = 180

Residual Error ($E_R$) = 180 – 151 = 29

**Reliability Factor (RF) = 1- 29/180 = .83**

**Table 1: Comparison of RF at Threshold time and after completing the testing time**

| Time (in Months | Reliability Factor (RF) |
|---|---|
| 94 Months | .83 |
| 109 Months | .88 |

**Mozilla Thunderbird:** Mozilla Thunderbird is a free [8], open source, cross-platform email and news client developed by the Mozilla Foundation. The project strategy is modeled after Mozilla Firefox, a project aimed at creating a web browser. On December 7, 2004, version 1.0 was released, and received over 500,000 downloads in its first three days of release, and 1,000,000 in 10 days [9] [10].

The bug report available on the official website of Apache OOo Bugzilla has been traced to find the bug report generated by the user [7]

**Calculating the values of b and α**

$$b` = \frac{\sum u \sum t - n\sum t u}{n \sum t2 - (\sum t)2} \quad \textbf{(from 7)}$$

n = 108

$\sum tu = -4943.69$ (From Table 4)

$\sum u = -62.0767$ (From Table 4)

$\sum t = 5886$ (From Table 4)

$\sum t2 = 425754$

$(\sum t)2 = 34644996$

$$b` = \frac{-62.0767 * 5886 - (108 * -4943.69)}{108 * 425754 - (5886)2}$$

$$b` = \frac{899301.52}{11336436}$$

**b` = 0.07932**

$$\alpha = \frac{-1.65321 + 0.07932842 * 108}{0.07932842} = \frac{6.91425936}{0.07932842}$$

**α = 87. 15**

**α = 87 months**

The predicted errors at 87 months = 217 (From Table 4)

And the actual errors found out and rectified is 206

Total Expected Error ($E_E$) = 225

Residual Error ($E_R$) = 225 – 217 = 8

**Reliability Factor (RF) = 1- 8/225 = 0.9644**

**Table 2: Comparison of RF at Threshold time and after completing the testing time**

| Time (in Months) | Reliability Factor (RF) |
|---|---|
| 87 Months | .96 |
| 108 Months | .98 |

In both the examples it is clearly seen that more than 90 percent of the errors has been detected by the threshold time (α). Therefore during the testing period after reaching the threshold time (α) the software is attaining reliability and further testing will enhance the reliability of the software to a greater level. Further the reliability of the software can be calculated by the Reliability Factor.

**Conclusion:** Thus the Reliability Factor (RF) of the Software Reliability Growth Model (SRGM) clearly depicts the authenticity or the reliability of the software under consideration. The value b` will always lie between 0 and 1. If the value of RF approaches 0 then the Modified G-O Model predicts that the number of residual errors is high and the reliability of the software is highly vulnerable. But at the end of the testing period, as the errors are being detected and rectified, the number of residual errors decreases with respect of total Expected Errors. As the residual errors decreases the reliability of the software goes on increasing. And it is being clearly depicted by the RF.

**Appendix A**

$\mu(t) = E_E (1 - e^{-bt})$ (From Original Goel-Okumoto Model)

therefore,

$$\frac{\mu(t)}{E_E} = (1 - e^{-bt})$$

$$e^{-bt} = 1 - \frac{\mu(t)}{E_E}$$

Taking natural log (ln) on both sides, we have

$$\ln(e^{-bt}) = \ln\left(1 - \frac{\mu(t)}{E_E}\right)$$

$$-bt = \ln\left(1 - \frac{\mu(t)}{E_E}\right) \quad \text{(because } \ln(e^x) = x\text{)}$$

and

$$\mathbf{b} = \frac{-\ln\left(1 - \frac{\mu(t)}{E_E}\right)}{t}$$

**Table 3: Reliability calculation of Debian Operating System**

| Time (t) (in months) | Actual Errors Reported | G-O Model | | | | | |
|---|---|---|---|---|---|---|---|
| | | Predicted Errors μ(t) (G-O Model) $\mu(t)=a\,(1-e^{-bt})$ | Predicted Errors (Rounded Off) | Residual Errors | u | t^2 | tu |
| 1 | 3 | 3.493291685 | 3 | 177 | -0.008511 | 1 | -0.008511 |
| 2 | 5 | 6.918788443 | 7 | 173 | -0.017023 | 4 | -0.034045 |
| 3 | 6 | 10.27780598 | 10 | 170 | -0.025534 | 9 | -0.076602 |
| 4 | 6 | 13.57163448 | 14 | 166 | -0.034045 | 16 | -0.136181 |
| 5 | 10 | 16.80153906 | 17 | 163 | -0.042556 | 25 | -0.212782 |
| 6 | 12 | 19.96876032 | 20 | 160 | -0.051068 | 36 | -0.306406 |
| 7 | 18 | 23.07451476 | 23 | 157 | -0.059579 | 49 | -0.417053 |
| 8 | 24 | 26.11999528 | 26 | 154 | -0.06809 | 64 | -0.544723 |
| 9 | 27 | 29.10637161 | 29 | 151 | -0.076602 | 81 | -0.689414 |
| 10 | 27 | 32.03479082 | 32 | 148 | -0.085113 | 100 | -0.851129 |
| 11 | 31 | 34.90637768 | 35 | 145 | -0.093624 | 121 | -1.029866 |
| 12 | 36 | 37.72223515 | 38 | 142 | -0.102135 | 144 | -1.225626 |
| 13 | 40 | 40.48344478 | 40 | 140 | -0.110647 | 169 | -1.438408 |
| 14 | 41 | 43.19106712 | 43 | 137 | -0.119158 | 196 | -1.668213 |
| 15 | 46 | 45.84614217 | 46 | 134 | -0.127669 | 225 | -1.91504 |
| 16 | 48 | 48.4496897 | 48 | 132 | -0.136181 | 256 | -2.17889 |
| 17 | 49 | 51.00270973 | 51 | 129 | -0.144692 | 289 | -2.459763 |
| 18 | 52 | 53.50618285 | 54 | 126 | -0.153203 | 324 | -2.757658 |
| 19 | 56 | 55.96107062 | 56 | 124 | -0.161714 | 361 | -3.072575 |
| 20 | 57 | 58.36831596 | 58 | 122 | -0.170226 | 400 | -3.404516 |
| 21 | 60 | 60.72884346 | 61 | 119 | -0.178737 | 441 | -3.753479 |
| 22 | 61 | 63.04355979 | 63 | 117 | -0.187248 | 484 | -4.119464 |
| 23 | 62 | 65.31335402 | 65 | 115 | -0.19576 | 529 | -4.502472 |
| 24 | 65 | 67.53909794 | 68 | 112 | -0.204271 | 576 | -4.902503 |
| 25 | 67 | 69.72164647 | 70 | 110 | -0.212782 | 625 | -5.319556 |
| 26 | 70 | 71.86183788 | 72 | 108 | -0.221294 | 676 | -5.753632 |
| 27 | 73 | 73.96049423 | 74 | 106 | -0.229805 | 729 | -6.20473 |
| 28 | 78 | 76.01842159 | 76 | 104 | -0.238316 | 784 | -6.672851 |
| 29 | 78 | 78.03641038 | 78 | 102 | -0.246827 | 841 | -7.157994 |
| 30 | 80 | 80.01523571 | 80 | 100 | -0.255339 | 900 | -7.66016 |
| 31 | 83 | 81.95565763 | 82 | 98 | -0.26385 | 961 | -8.179349 |
| 32 | 84 | 83.85842145 | 84 | 96 | -0.272361 | 1024 | -8.71556 |
| 33 | 86 | 85.72425798 | 86 | 94 | -0.280873 | 1089 | -9.268794 |
| 34 | 86 | 87.5538839 | 88 | 92 | -0.289384 | 1156 | -9.83905 |

| Time (t) (in months) | Actual Errors Reported | G-O Model | | | | | |
|---|---|---|---|---|---|---|---|
| | | Predicted Errors μ(t) (G-O Model) $\mu(t)=a(1-e^{-bt})$ | Predicted Errors (Rounded Off) | Residual Errors | u | t^2 | tu |
| 35 | 87 | 89.34800195 | 89 | 91 | -0.297895 | 1225 | -10.42633 |
| 36 | 90 | 91.10730124 | 91 | 89 | -0.306406 | 1296 | -11.03063 |
| 37 | 92 | 92.83245749 | 93 | 87 | -0.314918 | 1369 | -11.65196 |
| 38 | 94 | 94.52413333 | 95 | 85 | -0.323429 | 1444 | -12.2903 |
| 39 | 94 | 96.18297852 | 96 | 84 | -0.33194 | 1521 | -12.94567 |
| 40 | 94 | 97.80963021 | 98 | 82 | -0.340452 | 1600 | -13.61806 |
| 41 | 94 | 99.40471318 | 99 | 81 | -0.348963 | 1681 | -14.30748 |
| 42 | 99 | 100.9688401 | 101 | 79 | -0.357474 | 1764 | -15.01391 |
| 43 | 102 | 102.5026117 | 103 | 77 | -0.365985 | 1849 | -15.73737 |
| 44 | 103 | 104.0066172 | 104 | 76 | -0.374497 | 1936 | -16.47786 |
| 45 | 105 | 105.4814341 | 105 | 75 | -0.383008 | 2025 | -17.23536 |
| 46 | 107 | 106.9276291 | 107 | 73 | -0.391519 | 2116 | -18.00989 |
| 47 | 112 | 108.3457574 | 108 | 72 | -0.400031 | 2209 | -18.80144 |
| 48 | 119 | 109.7363639 | 110 | 70 | -0.408542 | 2304 | -19.61001 |
| 49 | 125 | 111.0999827 | 111 | 69 | -0.417053 | 2401 | -20.43561 |
| 50 | 127 | 112.4371375 | 112 | 68 | -0.425564 | 2500 | -21.27822 |
| 51 | 128 | 113.7483418 | 114 | 66 | -0.434076 | 2601 | -22.13786 |
| 52 | 131 | 115.0340994 | 115 | 65 | -0.442587 | 2704 | -23.01453 |
| 53 | 131 | 116.2949041 | 116 | 64 | -0.451098 | 2809 | -23.90821 |
| 54 | 132 | 117.5312401 | 118 | 62 | -0.45961 | 2916 | -24.81892 |
| 55 | 133 | 118.7435823 | 119 | 61 | -0.468121 | 3025 | -25.74665 |
| 56 | 137 | 119.9323964 | 120 | 60 | -0.476632 | 3136 | -26.6914 |
| 57 | 140 | 121.0981389 | 121 | 59 | -0.485143 | 3249 | -27.65318 |
| 58 | 140 | 122.2412577 | 122 | 58 | -0.493655 | 3364 | -28.63198 |
| 59 | 143 | 123.3621918 | 123 | 57 | -0.502166 | 3481 | -29.6278 |
| 60 | 147 | 124.4613717 | 124 | 56 | -0.510677 | 3600 | -30.64064 |
| 61 | 147 | 125.5392197 | 126 | 54 | -0.519189 | 3721 | -31.67051 |
| 62 | 153 | 126.5961496 | 127 | 53 | -0.5277 | 3844 | -32.7174 |
| 63 | 153 | 127.6325675 | 128 | 52 | -0.536211 | 3969 | -33.78131 |
| 64 | 153 | 128.6488715 | 129 | 51 | -0.544723 | 4096 | -34.86224 |
| 65 | 155 | 129.6454519 | 130 | 50 | -0.553234 | 4225 | -35.9602 |
| 66 | 155 | 130.6226915 | 131 | 49 | -0.561745 | 4356 | -37.07518 |
| 67 | 159 | 131.5809656 | 132 | 48 | -0.570256 | 4489 | -38.20718 |
| 68 | 163 | 132.5206423 | 133 | 47 | -0.578768 | 4624 | -39.3562 |
| 69 | 164 | 133.4420826 | 133 | 47 | -0.587279 | 4761 | -40.52225 |
| 70 | 167 | 134.3456403 | 134 | 46 | -0.59579 | 4900 | -41.70532 |

| Time (t) (in months) | Actual Errors Reported | G-O Model | | | | | |
|---|---|---|---|---|---|---|---|
| | | Predicted Errors μ(t) (G-O Model) $\mu(t)=a(1-e^{-bt})$ | Predicted Errors (Rounded Off) | Residual Errors | u | t^2 | tu |
| 71 | 170 | 135.2316625 | 135 | 45 | -0.604302 | 5041 | -42.90541 |
| 72 | 171 | 136.1004895 | 136 | 44 | -0.612813 | 5184 | -44.12252 |
| 73 | 171 | 136.952455 | 137 | 43 | -0.621324 | 5329 | -45.35666 |
| 74 | 171 | 137.7878863 | 138 | 42 | -0.629835 | 5476 | -46.60782 |
| 75 | 171 | 138.6071042 | 139 | 41 | -0.638347 | 5625 | -47.876 |
| 76 | 171 | 139.4104234 | 139 | 41 | -0.646858 | 5776 | -49.16121 |
| 77 | 171 | 140.1981525 | 140 | 40 | -0.655369 | 5929 | -50.46343 |
| 78 | 171 | 140.9705939 | 141 | 39 | -0.663881 | 6084 | -51.78268 |
| 79 | 171 | 141.7280445 | 142 | 38 | -0.672392 | 6241 | -53.11896 |
| 80 | 171 | 142.4707951 | 142 | 38 | -0.680903 | 6400 | -54.47225 |
| 81 | 171 | 143.199131 | 143 | 37 | -0.689414 | 6561 | -55.84257 |
| 82 | 171 | 143.9133319 | 144 | 36 | -0.697926 | 6724 | -57.22991 |
| 83 | 171 | 144.6136722 | 145 | 35 | -0.706437 | 6889 | -58.63427 |
| 84 | 171 | 145.3004209 | 145 | 35 | -0.714948 | 7056 | -60.05566 |
| 85 | 171 | 145.9738418 | 146 | 34 | -0.72346 | 7225 | -61.49406 |
| 86 | 171 | 146.6341934 | 147 | 33 | -0.731971 | 7396 | -62.9495 |
| 87 | 171 | 147.2817295 | 147 | 33 | -0.740482 | 7569 | -64.42195 |
| 88 | 171 | 147.9166987 | 148 | 32 | -0.748993 | 7744 | -65.91142 |
| 89 | 171 | 148.539345 | 149 | 31 | -0.757505 | 7921 | -67.41792 |
| 90 | 171 | 149.1499075 | 149 | 31 | -0.766016 | 8100 | -68.94144 |
| 91 | 171 | 149.7486206 | 150 | 30 | -0.774527 | 8281 | -70.48199 |
| 92 | 171 | 150.3357145 | 150 | 30 | -0.783039 | 8464 | -72.03955 |
| 93 | 171 | 150.9114145 | 151 | 29 | -0.79155 | 8649 | -73.61414 |
| 94 | 171 | 151.4759418 | 151 | 29 | -0.800061 | 8836 | -75.20575 |
| 95 | 171 | 152.0295132 | 152 | 28 | -0.808572 | 9025 | -76.81439 |
| 96 | 171 | 152.5723414 | 153 | 27 | -0.817084 | 9216 | -78.44004 |
| 97 | 171 | 153.1046348 | 153 | 27 | -0.825595 | 9409 | -80.08272 |
| 98 | 171 | 153.6265979 | 154 | 26 | -0.834106 | 9604 | -81.74242 |
| 99 | 171 | 154.1384311 | 154 | 26 | -0.842618 | 9801 | -83.41915 |
| 100 | 171 | 154.6403311 | 155 | 25 | -0.851129 | 10000 | -85.11289 |
| 101 | 171 | 155.1324907 | 155 | 25 | -0.85964 | 10201 | -86.82366 |
| 102 | 171 | 155.6150988 | 156 | 24 | -0.868152 | 10404 | -88.55145 |
| 103 | 171 | 156.0883409 | 156 | 24 | -0.876663 | 10609 | -90.29627 |
| 104 | 171 | 156.5523987 | 157 | 23 | -0.885174 | 10816 | -92.0581 |
| 105 | 171 | 157.0074504 | 157 | 23 | -0.893685 | 11025 | -93.83696 |
| 106 | 171 | 157.4536709 | 157 | 23 | -0.902197 | 11236 | -95.63285 |

| Time (t) (in months) | Actual Errors Reported | G-O Model | | | | | |
|---|---|---|---|---|---|---|---|
| | | Predicted Errors μ(t) (G-O Model) $\mu(t)=a\,(1-e^{-bt})$ | Predicted Errors (Rounded Off) | Residual Errors | u | t^2 | tu |
| 107 | 171 | 157.8912314 | 158 | 22 | -0.910708 | 11449 | -97.44575 |
| 108 | 171 | 158.3203002 | 158 | 22 | -0.919219 | 11664 | -99.27568 |
| 109 | 172 | 158.7410419 | 159 | 21 | -0.927731 | 11881 | -101.1226 |
| **5995** | | | | | **-51.025** | **437635** | **-3724.8** |

**Table 4: Reliability calculation of Thunderbird**

| Time (t) (in months) | Actual Errors Reported ($E_A(t)$) | Predicted Errors μ(t) (G-O Model) $\mu(t)=a\,(1-e^{-bt})$ | Predicted Errors μ(t) (Rounded Off) | Residual Errors | u | t^2 | tu |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 8.388735 | 8 | 217 | -0.00193 | 1 | -0.00193 |
| 2 | 2 | 16.46471 | 16 | 209 | -0.00388 | 4 | -0.00776 |
| 3 | 3 | 24.23959 | 24 | 201 | -0.00583 | 9 | -0.01749 |
| 4 | 4 | 31.72459 | 32 | 193 | -0.00779 | 16 | -0.03116 |
| 5 | 8 | 38.93053 | 39 | 186 | -0.01572 | 25 | -0.07861 |
| 6 | 10 | 45.86781 | 46 | 179 | -0.01974 | 36 | -0.11846 |
| 7 | 16 | 52.54644 | 53 | 172 | -0.03204 | 49 | -0.22425 |
| 8 | 19 | 58.97607 | 59 | 166 | -0.03832 | 64 | -0.30652 |
| 9 | 20 | 65.16599 | 65 | 160 | -0.04043 | 81 | -0.36386 |
| 10 | 25 | 71.12512 | 71 | 154 | -0.05115 | 100 | -0.51153 |
| 11 | 26 | 76.86208 | 77 | 148 | -0.05333 | 121 | -0.58662 |
| 12 | 30 | 82.38514 | 82 | 143 | -0.06215 | 144 | -0.74577 |
| 13 | 34 | 87.70229 | 88 | 137 | -0.07115 | 169 | -0.92494 |
| 14 | 42 | 92.8212 | 93 | 132 | -0.08973 | 196 | -1.25624 |
| 15 | 46 | 97.74926 | 98 | 127 | -0.09933 | 225 | -1.48994 |
| 16 | 49 | 102.4936 | 102 | 123 | -0.10667 | 256 | -1.70672 |
| 17 | 58 | 107.061 | 107 | 118 | -0.12947 | 289 | -2.20092 |
| 18 | 59 | 111.4582 | 111 | 114 | -0.13207 | 324 | -2.37734 |
| 19 | 62 | 115.6914 | 116 | 109 | -0.13999 | 361 | -2.6599 |
| 20 | 65 | 119.7668 | 120 | 105 | -0.14806 | 400 | -2.96125 |
| 21 | 68 | 123.6902 | 124 | 101 | -0.15628 | 441 | -3.28194 |
| 22 | 70 | 127.4674 | 127 | 98 | -0.16185 | 484 | -3.56072 |
| 23 | 72 | 131.1037 | 131 | 94 | -0.16749 | 529 | -3.8523 |
| 24 | 73 | 134.6045 | 135 | 90 | -0.17034 | 576 | -4.08813 |
| 25 | 76 | 137.9747 | 138 | 87 | -0.179 | 625 | -4.47491 |
| 26 | 79 | 141.2193 | 141 | 84 | -0.18783 | 676 | -4.88357 |
| 27 | 81 | 144.3429 | 144 | 81 | -0.19382 | 729 | -5.23314 |
| 28 | 84 | 147.3501 | 147 | 78 | -0.20296 | 784 | -5.68298 |

| Time (t) (in months) | Actual Errors Reported ($E_A(t)$) | Predicted Errors μ(t) (G-O Model) $\mu(t)=a(1-e^{-bt})$ | Predicted Errors μ(t) (Rounded Off) | Residual Errors | u | t^2 | tu |
|---|---|---|---|---|---|---|---|
| 29 | 87 | 150.2451 | 150 | 75 | -0.2123 | 841 | -6.1568 |
| 30 | 88 | 153.0322 | 153 | 72 | -0.21546 | 900 | -6.46386 |
| 31 | 89 | 155.7154 | 156 | 69 | -0.21864 | 961 | -6.77795 |
| 32 | 92 | 158.2986 | 158 | 67 | -0.22833 | 1024 | -7.30659 |
| 33 | 93 | 160.7854 | 161 | 64 | -0.23161 | 1089 | -7.64308 |
| 34 | 99 | 163.1795 | 163 | 62 | -0.25181 | 1156 | -8.56161 |
| 35 | 101 | 165.4844 | 165 | 60 | -0.25876 | 1225 | -9.05663 |
| 36 | 103 | 167.7033 | 168 | 57 | -0.26582 | 1296 | -9.56962 |
| 37 | 105 | 169.8396 | 170 | 55 | -0.273 | 1369 | -10.101 |
| 38 | 106 | 171.8961 | 172 | 53 | -0.27664 | 1444 | -10.5122 |
| 39 | 107 | 173.876 | 174 | 51 | -0.2803 | 1521 | -10.9317 |
| 40 | 108 | 175.7821 | 176 | 49 | -0.284 | 1600 | -11.3599 |
| 41 | 110 | 177.6171 | 178 | 47 | -0.29148 | 1681 | -11.9509 |
| 42 | 113 | 179.3837 | 179 | 46 | -0.30296 | 1764 | -12.7245 |
| 43 | 115 | 181.0844 | 181 | 44 | -0.31079 | 1849 | -13.364 |
| 44 | 117 | 182.7217 | 183 | 42 | -0.31876 | 1936 | -14.0254 |
| 45 | 117 | 184.298 | 184 | 41 | -0.31876 | 2025 | -14.3441 |
| 46 | 122 | 185.8155 | 186 | 39 | -0.33935 | 2116 | -15.6099 |
| 47 | 124 | 187.2764 | 187 | 38 | -0.34786 | 2209 | -16.3495 |
| 48 | 127 | 188.6829 | 189 | 36 | -0.36096 | 2304 | -17.3259 |
| 49 | 129 | 190.0369 | 190 | 35 | -0.36991 | 2401 | -18.1257 |
| 50 | 131 | 191.3404 | 191 | 34 | -0.37905 | 2500 | -18.9527 |
| 51 | 133 | 192.5954 | 193 | 32 | -0.38839 | 2601 | -19.8081 |
| 52 | 137 | 193.8035 | 194 | 31 | -0.4077 | 2704 | -21.2004 |
| 53 | 137 | 194.9666 | 195 | 30 | -0.4077 | 2809 | -21.6081 |
| 54 | 137 | 196.0864 | 196 | 29 | -0.4077 | 2916 | -22.0158 |
| 55 | 139 | 197.1644 | 197 | 28 | -0.41768 | 3025 | -22.9726 |
| 56 | 142 | 198.2022 | 198 | 27 | -0.4331 | 3136 | -24.2538 |
| 57 | 142 | 199.2013 | 199 | 26 | -0.4331 | 3249 | -24.687 |
| 58 | 143 | 200.1631 | 200 | 25 | -0.43837 | 3364 | -25.4254 |
| 59 | 146 | 201.0891 | 201 | 24 | -0.45456 | 3481 | -26.8188 |
| 60 | 148 | 201.9806 | 202 | 23 | -0.46569 | 3600 | -27.9415 |
| 61 | 149 | 202.8389 | 203 | 22 | -0.47137 | 3721 | -28.7535 |
| 62 | 149 | 203.6651 | 204 | 21 | -0.47137 | 3844 | -29.2249 |
| 63 | 149 | 204.4605 | 204 | 21 | -0.47137 | 3969 | -29.6962 |
| 64 | 151 | 205.2263 | 205 | 20 | -0.48295 | 4096 | -30.9089 |
| 65 | 153 | 205.9635 | 206 | 19 | -0.49485 | 4225 | -32.1653 |
| 66 | 158 | 206.6733 | 207 | 18 | -0.52611 | 4356 | -34.7231 |

| Time (t) (in months) | Actual Errors Reported ($E_A(t)$) | Predicted Errors μ(t) (G-O Model) $\mu(t)=a(1-e^{-bt})$ | Predicted Errors μ(t) (Rounded Off) | Residual Errors | u | t^2 | tu |
|---|---|---|---|---|---|---|---|
| 67 | 159 | 207.3566 | 207 | 18 | -0.53264 | 4489 | -35.6868 |
| 68 | 160 | 208.0144 | 208 | 17 | -0.53927 | 4624 | -36.6703 |
| 69 | 162 | 208.6476 | 209 | 16 | -0.55284 | 4761 | -38.1461 |
| 70 | 162 | 209.2573 | 209 | 16 | -0.55284 | 4900 | -38.6989 |
| 71 | 164 | 209.8442 | 210 | 15 | -0.56685 | 5041 | -40.2465 |
| 72 | 167 | 210.4093 | 210 | 15 | -0.58875 | 5184 | -42.3903 |
| 73 | 169 | 210.9533 | 211 | 14 | -0.60399 | 5329 | -44.0916 |
| 74 | 171 | 211.477 | 211 | 14 | -0.61979 | 5476 | -45.8644 |
| 75 | 172 | 211.9812 | 212 | 13 | -0.62791 | 5625 | -47.093 |
| 76 | 174 | 212.4666 | 212 | 13 | -0.64461 | 5776 | -48.9905 |
| 77 | 178 | 212.9339 | 213 | 12 | -0.68008 | 5929 | -52.3665 |
| 78 | 185 | 213.3837 | 213 | 12 | -0.75012 | 6084 | -58.5096 |
| 79 | 189 | 213.8168 | 214 | 11 | -0.79588 | 6241 | -62.8745 |
| 80 | 192 | 214.2338 | 214 | 11 | -0.83367 | 6400 | -66.6935 |
| 81 | 198 | 214.6352 | 215 | 10 | -0.92082 | 6561 | -74.5863 |
| 82 | 200 | 215.0216 | 215 | 10 | -0.95424 | 6724 | -78.2479 |
| 83 | 201 | 215.3936 | 215 | 10 | -0.97197 | 6889 | -80.6736 |
| 84 | 204 | 215.7518 | 216 | 9 | -1.02996 | 7056 | -86.5169 |
| 85 | 204 | 216.0966 | 216 | 9 | -1.02996 | 7225 | -87.5469 |
| 86 | 205 | 216.4285 | 216 | 9 | -1.05115 | 7396 | -90.3991 |
| 87 | 206 | 216.7481 | 217 | 8 | -1.07343 | 7569 | -93.3883 |
| 88 | 209 | 217.0558 | 217 | 8 | -1.14806 | 7744 | -101.03 |
| 89 | 210 | 217.3519 | 217 | 8 | -1.17609 | 7921 | -104.672 |
| 90 | 210 | 217.6371 | 218 | 7 | -1.17609 | 8100 | -105.848 |
| 91 | 212 | 217.9116 | 218 | 7 | -1.23824 | 8281 | -112.68 |
| 92 | 212 | 218.1759 | 218 | 7 | -1.23824 | 8464 | -113.918 |
| 93 | 212 | 218.4303 | 218 | 7 | -1.23824 | 8649 | -115.156 |
| 94 | 214 | 218.6752 | 219 | 6 | -1.31079 | 8836 | -123.214 |
| 95 | 214 | 218.9111 | 219 | 6 | -1.31079 | 9025 | -124.525 |
| 96 | 214 | 219.1381 | 219 | 6 | -1.31079 | 9216 | -125.836 |
| 97 | 215 | 219.3566 | 219 | 6 | -1.35218 | 9409 | -131.162 |
| 98 | 216 | 219.567 | 220 | 5 | -1.39794 | 9604 | -136.998 |
| 99 | 218 | 219.7696 | 220 | 5 | -1.50708 | 9801 | -149.201 |
| 100 | 218 | 219.9646 | 220 | 5 | -1.50708 | 10000 | -150.708 |
| 101 | 219 | 220.1523 | 220 | 5 | -1.57403 | 10201 | -158.977 |
| 102 | 219 | 220.3331 | 220 | 5 | -1.57403 | 10404 | -160.551 |
| 103 | 220 | 220.5071 | 221 | 4 | -1.65321 | 10609 | -170.281 |
| 104 | 220 | 220.6746 | 221 | 4 | -1.65321 | 10816 | -171.934 |

| Time (t) (in months) | Actual Errors Reported ($E_A(t)$) | Predicted Errors μ(t) (G-O Model) $\mu(t)=a(1-e^{-bt})$ | Predicted Errors μ(t) (Rounded Off) | Residual Errors | u | t^2 | tu |
|---|---|---|---|---|---|---|---|
| 105 | 220 | 220.8358 | 221 | 4 | -1.65321 | 11025 | -173.587 |
| 106 | 220 | 220.9911 | 221 | 4 | -1.65321 | 11236 | -175.241 |
| 107 | 220 | 221.1406 | 221 | 4 | -1.65321 | 11449 | -176.894 |
| 108 | 220 | 221.2845 | 221 | 4 | -1.65321 | 11664 | -178.547 |
| **5886** | | | | | **-62.0767** | **425754** | **-4943.69** |

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